TRANSACTIONS

OF THE

SOUTH AFRICAN PHILOSOPHICAL SOCIETY.

NOTE ON THE THREE-POINT, OR POTHENOT'S, PROBLEM.

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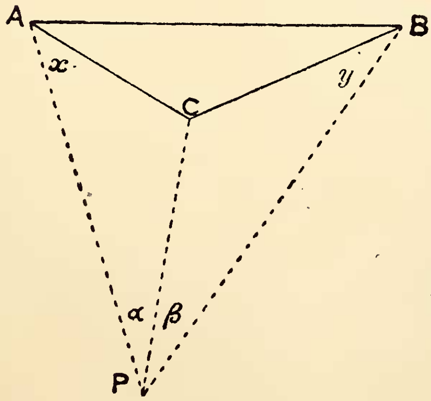
The ordinary methods of computation of the position of a point, given the angles subtended by three other points of known positions are chiefly :—

First method.



1. Compute the length and "angle of direction" of a from the co-ordinates of C and A.

2. Compute the length and angle of direction of β from C and B.

3. Put $PAC = x$, $PBC = y$. Then $\tan \frac{1}{2}(x-y) = \tan(z-45^\circ) \tan \frac{1}{2}(x+y)$ Where

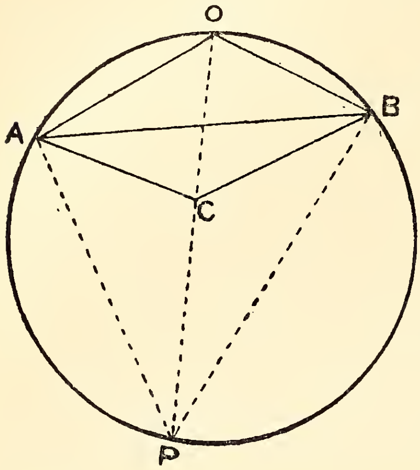
$$\tan z = \frac{a \sin \beta}{b \sin a}$$

and

$$\tfrac{1}{2}(x+y) = 180^\circ - \tfrac{1}{2}(a+\beta+C).$$

4. Compute the co-ordinates of P either from triangle P C A or triangle P B C.

Second method.



1. Compute the length and angle of direction of C from the co-ordinates of A and B.

2. Compute the co-ordinates of O from the triangle O A B in which O A B $=\beta$ and O B A $=\alpha$.

3. Compute the angle of direction of O C from the co-ordinates of O and C.

4. Compute the co-ordinates of P either from triangle P O A or P B O.

Both these methods are avoided by many surveyors on account of their length. A shorter method will now be given, with a numerical example showing the arrangement of the computation.

Taking the middle point C for origin, put x' y' and x'' y'' for the co-ordinates of A and B. The equations to the circles (1) through A and C and containing the angle α (2) through C and B and containing the angle β are

$$\tan\alpha \left\{ y\,(y-y')+x\,(x-x') \right\} - xy'+yx'=0$$
$$\tan\beta \left\{ y\,(y-y'')+x\,(x-x'') \right\} - yx''+xy''=0$$

reducible to

$$y^2+x^2+Ay-Bx=0$$
$$y^2+x^2-Cy+Dx=0$$

Where

$$A=x'\cot\alpha-y' \qquad B=y'\cot\alpha+x'$$
$$C=x''\cot\beta+y'' \qquad D=y''\cot\beta-x''$$

Then

$$\frac{y}{x}=\frac{B+D}{A+C}=m$$

$$m^2x+x=B-mA$$

$$x=\frac{B-mA}{m^2+1} \qquad y=mx$$

Example.

A −1811·59	−1018·55	$y' = -1376{\cdot}55$	$x' = +406{\cdot}90$
B + 6·81	−930·26	$y'' = +\ 441{\cdot}85$	$x'' = +495{\cdot}19$
C − 435·04	−1425·45	0·00	0·00

$\alpha = 64{\cdot}7{\cdot}40$

$\beta = 20{\cdot}33{\cdot}20$

+9·685719	+9·685719	+0·425980	+0·425980
+2·609488	−3·138792	+2·694772	+2·645275
+2·295207	−2·824511	+3·120752	+3·071255
+197·34	−667·59	+1320·54	+1178·30
$-y'$ +1376·55	$+x'$ +406·90	$+y''$ +441·85	$-x''$ −495·19
A+1573·89	B−260·69	C+1762·39	D+683·11
A+3·196974	−2·662730	A+1573·89	B−260·69
m9·102482	0·006906	+3336·28	+422·42
+2·299456	x −2·655824	2·625744	m9·102482
$-m$A−199·28	9 102482	3·523262	$m^2$8·204964
+B−260·69	y −1·758306	m^2+1	=1·01603
−459·97		y −57·32	x −452·71
		−435·04	−1425·45
	Co-ordinates of P :	−492·36	−1878·16

A check is afforded by the computation of the angles of direction P A and P B. P C is given by

$$\tan^{-1} m.$$

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